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HEAT TRANSFER IN A PLANE THREE-LAYER SYSTEM WITH A NOT-THROUGH CROSS CONNECTION

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The regularities of formation of temperature fields in a plane three-layer system with a not-through cross connection (connector) are considered with boundary conditions of the second kind on one exterior surface of the system. The character of distribution of the temperature fields in the zone of influence of the steel connector is investigated experimentally. The results of the experiment are compared to the numerical solution of the problem with boundary conditions of the second and third kind on the interior surface of the wall. Their satisfactory agreement is shown.

The regularities of heat transfer in a plane three-layer system with a not-through heat-conducting connection (connector) have been investigated theoretically in [1]. To extend this investigation it became necessary to solve the problem of heat conduction in a plane three-layer system with a not-through cross connection in the case of boundary conditions of the second kind on one exterior surface of the system for optimum designing of energy-saving enclosing structures of buildings and development of systems of their external thermal protection (warmth-keeping).

The present work seeks to develop an efficient numerical method of solution of the problem, which is rapidly adaptable to different configurations of multilayer external enclosures with cross connections, to compare results of numerical solution of the problem of heat transfer in a three-layer exterior wall with a connector in the case of boundary conditions of the second and third kind on its interior surface, to experimentally investigate the character of distribution of temperature fields in the zone of influence of a steel connector, and to compare the results of numerical solution to experimental data.

Physicomathematical Formulation and Method of Solution of the Problem. Heat transfer through a plane multilayer system with a cross connection will be considered using a brick three-layer exterior wall with a cylindrical connector as an example (Fig. 1). The internal and external layers of the enclosure represent the brickwork and the central layer is a warmth-keeping jacket. The ends of the connector are embedded in the internal and external layers of the enclosure. The geometric dimensions of the layers of the enclosure and the connector are prescribed. The thermophysical characteristics of the wall material (λ_i, ρ_i, and c_i, $i = \overline{1, 4}$), which are generally dependent on temperature, are known. The temperature of the medium $t_{\mathrm{g,e}}$ and the heat-transfer coefficient α_{w} are prescribed on the exterior surface of the enclosure, and the heat-flux density q_0 is prescribed on the interior surface. The temperature profile over the enclosure thickness t_0, t_{12}, t_{23}, and t_{w} outside the zone of influence of the connector is determined from the known value of q_0 from the analytical solution of a one-dimensional stationary heat-conduction problem [2].

We will solve the problem formulated in a cylindrical coordinate system (Fig. 1). The origin of coordinates will be located on the interior surface of the wall. The x axis will be guided normally to the wall, whereas the r axis will be guided along it. The connector axis coincides with the x axis. In numerical solution, we replace the mathematical domain of definition of the problem $\{0 \le x \le \delta, 0 \le r < \infty, 0 \le \tau \le \tau_{\mathrm{f}}\}$ by a closed computational domain $\overline{D}\{0 \le x \le X_{\mathrm{f}}, 0 \le r \le R_{\mathrm{f}}, 0 \le \tau \le \tau_{\mathrm{f}}\}$. Here, δ_1, δ_2, and δ_3 are the thicknesses of the first, second, and third layers of the wall and $\delta = \delta_1 + \delta_2 + \delta_3$ is the total wall thickness. The computational domain $\overline{D}$ will be subdivided into four subdomains: 1–3 are the internal, central, and external wall layers without a connector and 4 is the connector. Heat transfer in each of the subdomains in question is described by the two-dimensional, nonlinear, nonstationary heat-conduction equation

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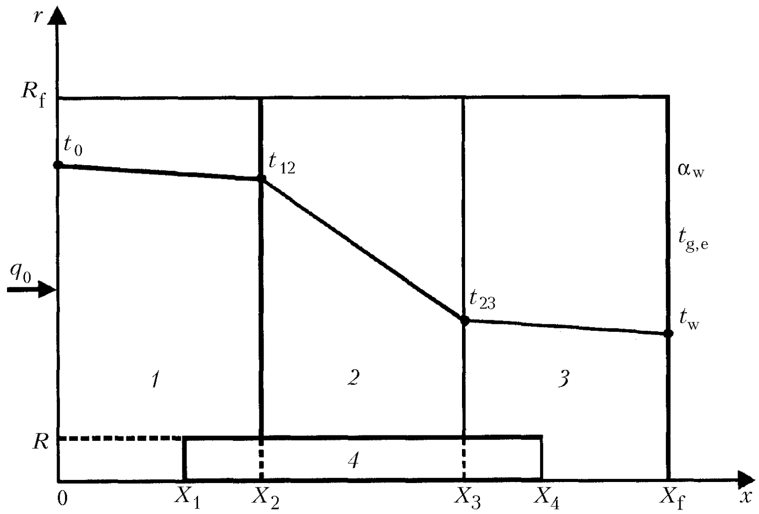


Fig. 1. Diagram of the three-layer external enclosure with a connector: 1, 2, and 3) internal, central, and external layers of the enclosure; 4) connector.

$$(\rho c)_i \frac{\partial t_i}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_i \frac{\partial t_i}{\partial r}\right) + \frac{\partial}{\partial x}\left(\lambda_i \frac{\partial t_i}{\partial x}\right), \quad i = \overline{1, 4}, \quad x, r, \tau \in D . \tag{1}$$

The system of equations (1) is closed by the initial and boundary conditions:

$$t_i \big|_{\tau=0} = t_i(x), \quad x \in \overline{D}, \quad i = \overline{1, 4}; \tag{2}$$

$$-\lambda_1 \frac{\partial t_1}{\partial x}\bigg|_{x=0} = q_0, \quad 0 \le r \le R_f; \tag{3}$$

$$\lambda_3 \frac{\partial t_3}{\partial x}\bigg|_{x=X_f} = \alpha_w (t_{g,e} - t_w), \qquad 0 \le r \le R_f; \tag{4}$$

$$\frac{\partial t_i}{\partial r}\bigg|_{r=0} = 0, \quad i = 1, 3, 4, \quad 0 \le x \le X_f; \tag{5}$$

$$\frac{\partial t_i}{\partial r}\bigg|_{r=R_f} = 0, \quad i = \overline{1, 3}, \quad 0 \le x \le X_f; \tag{6}$$

$$t_1 \big|_{x=X_1} = t_4 \big|_{x=X_1}, \quad \lambda_1 \frac{\partial t_1}{\partial x}\bigg|_{x=X_1} = \lambda_4 \frac{\partial t_4}{\partial x}\bigg|_{x=X_1}, \quad 0 \le r \le R; \tag{7}$$

$$t_4 \big|_{x=X_4} = t_3 \big|_{x=X_4}, \quad \lambda_4 \frac{\partial t_4}{\partial x}\bigg|_{x=X_4} = \lambda_3 \frac{\partial t_3}{\partial x}\bigg|_{x=X_4}, \quad 0 \le r \le R; \tag{8}$$

$$t_1\big|_{x=X_2} = t_2\big|_{x=X_2}\,, \quad \lambda_1 \frac{\partial t_1}{\partial x}\bigg|_{x=X_2} = \lambda_2 \frac{\partial t_2}{\partial x}\bigg|_{x=X_2}\,, \quad R \le r \le R_{\mathrm{f}}\,; \tag{9}$$

$$t_2\big|_{x=X_3} = t_3\big|_{x=X_3}\,, \quad \lambda_2 \frac{\partial t_2}{\partial x}\bigg|_{x=X_3} = \lambda_3 \frac{\partial t_3}{\partial x}\bigg|_{x=X_3}\,, \quad R \le r \le R_{\mathrm{f}}\,; \tag{10}$$

$$t_4\big|_{r=R} = t_1\big|_{r=R}\,, \quad \lambda_4 \frac{\partial t_4}{\partial r}\bigg|_{r=R} = \lambda_1 \frac{\partial t_1}{\partial r}\bigg|_{r=R}\,, \quad X_1 \le x \le X_2\,; \tag{11}$$

$$t_4\big|_{r=R} = t_2\big|_{r=R}\,, \quad \lambda_4 \frac{\partial t_4}{\partial r}\bigg|_{r=R} = \lambda_2 \frac{\partial t_2}{\partial r}\bigg|_{r=R}\,, \quad X_2 \le x \le X_3\,; \tag{12}$$

$$t_4\big|_{r=R} = t_3\big|_{r=R}\,, \quad \lambda_4 \frac{\partial t_4}{\partial r}\bigg|_{r=R} = \lambda_3 \frac{\partial t_3}{\partial r}\bigg|_{r=R}\,, \quad X_3 \le x \le X_4\,, \quad \tau \in [0, \tau_{\mathrm{f}}]\,. \tag{13}$$

At the boundaries of the computational domain, boundary condition of the second kind (3) holds for $x = 0$, the condition of convective heat exchange (4) is specified for $x = X_{\mathrm{f}}$, symmetry condition (5) holds on the $r = 0$ axis, the condition of independence of the process of heat transfer from r (6) is specified at the periphery of the domain for $r = R_{\mathrm{f}}$, and conditions of fourth kind (7)–(13) hold at the internal boundaries of subdomains 1–4. In formula (1), $D\{0 < x < X_{\mathrm{f}},\ 0 < r < R_{\mathrm{f}},\ 0 < \tau \le \tau_{\mathrm{f}}\}$ is the open computational domain of the problem.

We used the splitting method of N. N. Yanenko [3] for numerical solution of the problem. One-dimensional equations (resulting from the splitting) of heat conduction in single-layer and multilayer domains in the directions x and r were calculated by the iteration-interpolation method [4]. Systems of nonlinear difference equations with three-diagonal matrices were solved by the marching method [5] with iterations by the coefficients with a prescribed accuracy. Special difference equations allowing for the difference in the thermophysical characteristics of the materials of the subdomains [6] were used at the internal boundaries of the domain d.

Numerical solution of the problem according to the algorithm presented above was realized using a program developed in the FORTRAN language. The module aufbau principle of the program was used. Individual software modules, such as solution of a parabolic-type equation of general form with boundary conditions of the first to fourth kind in single-layer and multilayer domains, solution of a system of algebraic equations with a three-diagonal matrix by the marching method, and some others, were tested using the known analytical solutions [5] and solutions obtained with the methods of trial functions. The module aufbau principle of the program makes it possible to rapidly adapt it to any configuration of a multilayer external enclosure with the number of layers larger than three and to any position of the connector. The initial data and calculation results were interpolated using cubic spline functions [7].

In the general case, we took variable computational space steps in the directions x and r because of the large difference in the thermophysical characteristics of the materials of the wall layers and the connector. The solution of the problem was checked on clustering space and time grids. The acceptable number of nodes of the difference grid in x and r turned out to be equal to 65×200 for the basic variant of calculation. The time step was taken to be 30 sec. With further duplication of the number of space and time points the solution of the problem differed by no more than 0.05%. The value of the quantity R_{f} modeling the upper boundary of the domain of definition of the problem in r was selected from the condition of coincidence of the numerical solution of the initial two-dimensional problem for $r = R_{\mathrm{f}}$ and $\tau = \tau_{\mathrm{f}}$ with the known analytical solution of the one-dimensional stationary problem from [2]. The counting time of the basic variant of the problem for a physical time of the process equal to 72 h turned out to be no longer than 5.5 min on a Pentium 3 personal computer.

To test the numerical algorithm and the program, we solved, first of all, a two-dimensional problem of heat exchange in the three-layer system without a connector at a constant temperature of the external medium and a constant density of the heat flux on one surface of the system. After the establishment of the stationary regime, the numerical solution of the two-dimensional problem was compared to the analytical solution of a one-dimensional linear stationary problem; the analytical solution was expressed by the calculated heat-transfer resistances:

$$t(x) = t_{g,ins} - \frac{R_{ext.w}(x)}{R_{ext.w}} (t_{g,ins} - t_{g,e}) , \tag{14}$$

where $R_{ext.w}(x)$ is the calculated resistance of the system to heat transfer from the heat flux to the plane with a coordinate x and $R_{ext.w}$ is the resistance of the entire system to heat transfer, calculated from the formula

$$R_{ext.w} = \frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3} + \frac{1}{\alpha_w} . \tag{15}$$

It was established from the results of numerical calculations that, irrespective of the initial condition specified, the numerical solution of the two-dimensional problem tends to a unique stationary solution coincident with the analytical solution (14), which is one confirmation of the reliability of the calculation results. In solving the two-dimensional problem with a connector, we selected the stationary temperature profile over x, obtained from the analytical solution of the one-dimensional problem of heat conduction in the three-layer system (14), as the initial condition. This is justified from the physical viewpoint and leads to a reduction in the counting time of the problem because of the acceleration of the heat-transfer regime reaching the steady state.

For laboratory investigation of the temperature fields we made a 570 × 630 × 600 mm fragment of a three-layer brick wall with an efficient warmth-keeping jacket. The thickness of the internal wall layer was 250 mm, that of the external layer was 120 mm, and the thickness of the warmth-keeping layer was 200 mm. A flexible metallic connection (connector) of diameter 4 mm was made from reinforcing wire and was installed at the center of the wall's fragment under study. The length of the connector was 300 mm, and the depth of its embedding in the internal and external wall layers was 50 mm.

A specially manufactured 600 × 600 mm plane heater with a controlled power level of 5–50 W and a supply voltage of 36 V was installed on the interior of the structure to create a directed heat flux and a temperature difference over its thickness. To reduce the heat loss through the lateral surfaces the structure was erected on a foamed-plastic sheet of thickness 0.2 m. The entire structure, except for its exterior surface, was wrapped in six layers of URSA warmth-keeping jacket with reflecting baffles from foil of total thickness 0.3 m.

A thermocouple wire of diameter 0.2 mm calibrated at the Science and Production Association of the D. I. Mendeleev All-Russia Research Institute of Metrology was used for manufacture of Chromel-Alumel thermocouples. All 50 thermocouples were tested for the identity of readings using a thermostat and a reference thermocouple, for which purpose hot junctions of the thermocouples under study and of the reference thermocouple were placed in drilled holes of metal ingots with the subsequent filling of the holes with tin. Thereafter we placed the metal ingots in the thermostat with a constant and uniform temperature. Cold junctions were immersed in a vessel with melting ice. The thermocouples were tested at temperatures of 0, 25, and 100^{o}C. From the results of these tests, we composed tables of experimental data, which showed the deviations of the reading of the operating thermocouples from those of the reference one [8].

The thermocouples were fixed on the interior and exterior wall surfaces. In the bulk of the wall, they were installed both on the axis of the connector and in the radial direction from it, at the junctures of the brickwork and the warmth-keeping jacket, and in the connector itself and on the connector surface. In measuring the heat-fluxes, we used heat-flux transformer (heat-flow meter) Nos. 259 and 95-10 manufactured at the "Farada" Science and Technology Center (Moscow); they were installed on the interior (No. 259) and exterior (No. 95-10) surfaces. A Tadiran TGI-18H conditioner was used to maintain the air temperature at the laboratory at a constant level.

After the unit reached the stationary regime of heat transfer, we performed 10 measurements of the temperatures and heat fluxes at an interval of 3 h. The air temperature in the room was maintained at a level of 19 $\pm$ 0.1^{o}C.

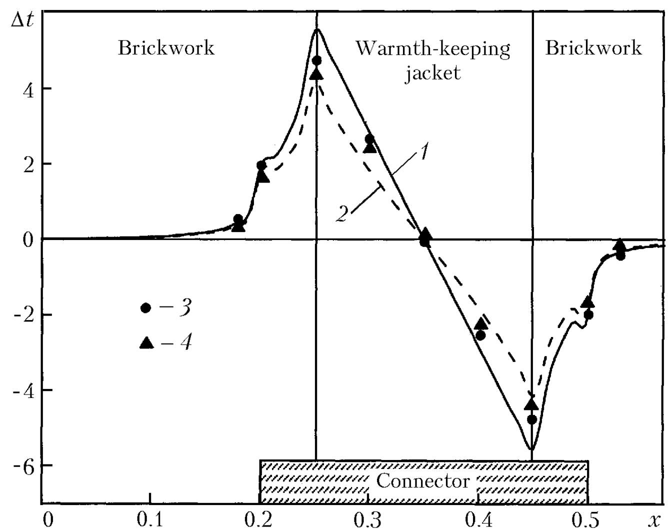


Fig. 2. Distribution of the temperature differences: 1) λ_2 = 0.001; 2) 0.05 W/(m·K); 3 and 4) upper and lower levels of readings of measurements. x, m; Δt, $^{\circ}$C.

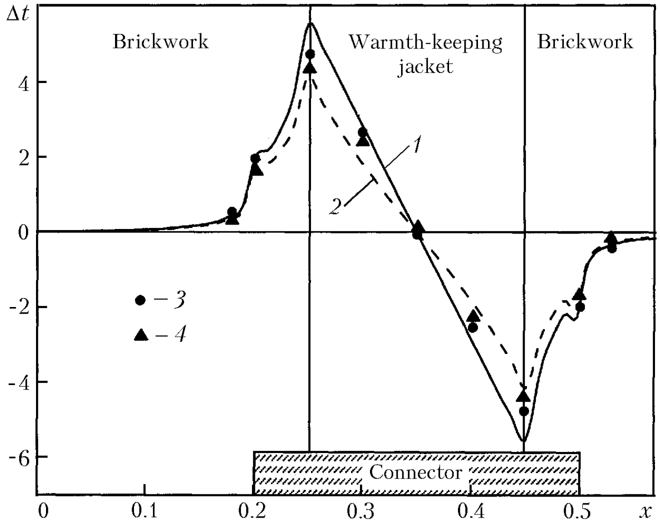
The temperature on the interior wall surface was 64.4$\pm$ 0.1°C and that on the exterior surface was 21 $\pm$ 0.1°C. The measured densities of the heat fluxes through the wall under test were equal to 9.3 $\pm$ 0.2 W/m^2 according to the readings of heat-flow meter No. 259 and to 8.7 $\pm$ 0.2 W/m^2 according to the readings of heat-flow meter No. 95-10. The relative measurement error was no larger than 7.1% for the temperatures and 8.8% for the heat fluxes.

Figure 2 gives results of a comparison of the numerical and laboratory experiments. Curve 1 characterizes the distribution of the temperature differences $\Delta t = t(x, r)\big|_{r=\infty} - t(x, r)\big|_{r=0}$ in different cross sections x, resulting from the numerical calculations for the thermal conductivity of the warmth-keeping jacket λ_2 = 0.0001 W/(m·K); also, the figure shows the limiting case of variation in the temperature differences on condition that there is no heat transfer between the connection and the warmth-keeping jacket, in practice. Thus, curve 1 characterizes the maximum temperature difference when the entire heat supplied to the interior wall surface reaches the exterior surface only through the connector. The disturbance of the temperature field is maximum at the sites of contact of the warmth-keeping jacket with the brickwork and is 5.5°C for this case. Curve 2 characterizes an analogous distribution of the temperature differences when an ideal thermal contact exists between the warmth-keeping jacket and the connector and the thermal conductivity of the warmth-keeping jacket is λ_2 = 0.05 W/(m·K). The disturbances of the temperature field in the characteristic cross sections x are about 4.1°C.

In solving the problem numerically for experimental conditions, we calculated the variant where boundary conditions of the third kind were specified on the exterior wall surfaces [1]. It was established that, with the prescribed initial data for which the heat-flux densities with boundary conditions of the second and third kind coincide, the calculated temperature fields in the wall with a connector are identical.

The results (presented in Fig. 2) of experimental investigations of the temperature fields (3 and 4 are the upper and lower levels of spread in experimental points respectively) confirm the presence of the largest disturbances of the temperature field (4.5 $\pm$ 0.2°C) at the sites of contact of the warmth-keeping jacket with the brickwork and the presence of a cross section (x = 0.35 m) with a maximum value of transmission heat [1] in the bulk of the wall; before this cross section, most of the heat arrives at the connector through the lateral surfaces. After this cross section, the heat in the zone of negative temperatures is removed from the connector to the materials of the exterior wall. The maximum spread in experimental points is no larger than 8.5%. The difference between the results of the experiment and those of the numerical calculations is within 14–21% for λ_2 = 0.0001 W/(m·K) and within 4–12% for λ_2 = 0.05 W/(m·K).

A comparison of the results of the numerical and laboratory experiments allows the conclusion on their satisfactory agreement.



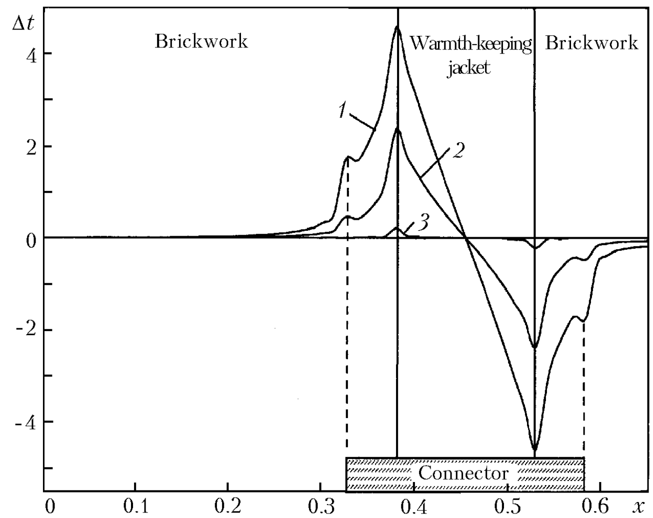


Fig. 3. Distribution of the temperature fields in the three-layer wall: 1) without a connector; 2, 3, and 4) with connectors having a thermal conductivity of λ_4 = 0.55, 20, and 58 W/(m·K) respectively.

Fig. 4. Distribution of the temperature differences Δt: 1) $\lambda_4 = 58$; 2) 20; 3) 0.55 W/(m·K). x, m; Δt, ^{o}C.

Figure 3 gives a plot characterizing the distribution of the temperature fields in the three-layer system with connectors having different thermophysical characteristics. Curve 1 shows the stationary temperature distribution [2]. Curve 2, coincident with curve 1, characterizes the temperature distribution in the case where a connector from carbon-filled plastic with a thermal conductivity of $\lambda_4 = 0.55$ W/(m·K) is used. Curves 3 and 4 characterize the temperature distribution in the wall with a connector made from stainless steel with a thermal conductivity of $\lambda_4 = 20$ W/(m·K) and with a connector from reinforcing wire with a thermal conductivity of $\lambda_4 = 58$ W/(m·K) respectively. As compared to the stationary temperature profile in the enclosure without a connector, the presence of the connector leads to a reduction in the temperature in the zone of its location before the cross section $x = 0.455$ m and to an increase in the temperature after this cross section. As is clear from Fig. 3, the largest influence on the temperature field is exerted by connectors with a higher thermal conductivity.

In the calculations, we used the following initial data: $\lambda_1 = \lambda_3 = 0.8$ W/(m·K), $\lambda_2 = 0.05$ W/(m·K), $\rho_1 = \rho_3 = 1800$ kg/m^3, $\rho_2 = 100$ kg/m^3, $c_1 = c_3 = 880$ J/(kg·K), $c_2 = 1500$ J/(kg·K), $X_1 = 0.325$ m, $X_2 = 0.38$ m, $X_3 = 0.53$ m, $X_4 = 0.585$ m, $X_f = 0.65$ m, $R_f = 0.2$ m, $R = 0.002$ m, $q_0 = 14.4$ W/m^2 (for $\alpha_0 = 8.7$ W/(m^2·K) and $t_{g,ins} = 20^{o}$C), $\alpha_w = 23$ W/(m^2·K), and $t_{g,e} = -40^{o}$C.

Figure 4 shows the influence of the temperature characteristics of the connector material on the distribution of the temperature differences $\Delta t = t(x, r)\big|_{r=\infty} - t(x, r)\big|_{r=0}$ in different cross sections x. Curve 1 corresponds to the connector made from reinforcing steel ($\lambda_4 = 58$ W/(m·K), $\rho_4 = 7850$ kg/m^3, and $c_4 = 482$ J/(kg·K)), curve 2 corresponds to the connector from stainless steel ($\lambda_4 = 20$ W/(m·K), $\rho_4 = 5000$ kg/m^3, and $c_4 = 800$ J/(kg·K)), and curve 3 corresponds to the connector from carbon-filled plastic ($\lambda_4 = 0.55$ W/(m·K), $\rho_4 = 1350$ kg/m^3, and $c_4 = 1062$ J/(kg·K)). The general trend toward formation of the temperature field of a wall enclosure with a connector corresponds to the conclusions of [1]. Most of the heat before the cross section $x = 0.455$ m arrives at the connector through lateral surfaces. Then, in the zone of negative temperatures, heat is removed from the connector through its lateral surfaces and the end to the materials of the exterior wall. The disturbances of the temperature field are the largest in the zones of contact of the internal ($x = 0.38$ m) and external ($x = 0.53$ m) wall layers with the warmth-keeping jacket. The disturbances of the temperature field near the ends of the connector are fewer in number than those at its lateral surface. The maximum disturbance is introduced by the connector made from reinforcing steel, whereas the minimum disturbance is introduced by the carbon-filled-plastic connector. The maximum temperature difference Δt for them is equal to 4.58 and 0.21^{o}C respectively. For the stainless-steel connector, we have $\Delta t = 2.36^{o}$C. Also, noteworthy is the difference in the qualitative behavior of the curves. For the carbon-filled-plastic connector, for example, breakpoints at the

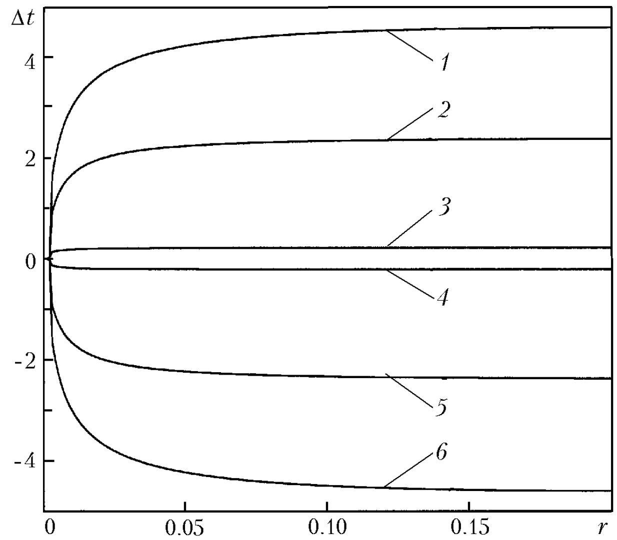


Fig. 5. Distribution of the temperature differences Δt: 1 and 6) λ_4 = 58; 2 and 5) 20; 3 and 4) 0.55 W/(m·K) [1, 2, and 3) x = 0.38; 4, 5, and 6) 0.53 m]. r, m; Δt, $^{\circ}$C.

junctures of the connector ends with the brickwork disappear, which is attributed to the proximity of the thermal conductivities of brick and carbon-filled plastic.

Figure 5 shows the distribution of the temperature differences $\Delta t = t(x, r) - t(x, r)\big|_{r=0}$ in the radial direction from the connector axis. It is seen that for x = 0.38 m and x = 0.53 m the zone of maximum influence on the temperature field of the wall of the connector made from stainless steel is about 0.03 m, and that of the reinforcing-wire connector is 0.06 m. The character of distribution of the temperature differences in the case of the carbon-filled-steel connector shows that the zone of its influence is no larger than the connector diameter.

Thus, from the comparison of the results of the numerical calculations obtained in this work and the experimental data, we can recommend that the above improved mathematical model, method of solution, and program of calculation of the temperature fields in a plane three-layer system with a not-through cross connection be used in practice. Using the program developed, we have established the character of distribution of the temperature fields in the wall with connectors having different thermophysical characteristics and have determined the maximum zones of their influence on the temperature field of the multilayer wall.

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NOTATION

c, specific heat, J/(kg·K); D, open computational domain of solution of the problem; $\overline{D}$, closed computational domain of solution of the problem; q, heat-flux density, W/m^2; r, x, cylindrical coordinates, m; R, connector radius, m; X_f and R_f, upper boundaries of computational domains in x and r, m; t, temperature, $^{\circ}$C; X_j ($j = \overline{1, 4}$), coordinates of the internal boundaries of computational domains in x, m; α, heat-transfer coefficient, W/(m^2·K); δ, total wall thickness, m; λ, thermal conductivity, W/(m·K); ρ, density, kg/m^3; τ, time, h. Subscripts: i, wall-material No.; j, No. of the internal boundaries of computational domains in x; in, initial state, g, air; ins, internal medium; e, external medium; 0, interior enclosure surface; w, exterior enclosure surface; f, finite quantity; ext.w, exterior wall; 1, 2, and 3, internal, central, and external layers of the enclosure; 4, connector.

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